The Market for Corporate Directors

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Abstract

This paper develops a matching model in the market for directors to explain equilibrium board quality. In my model, (1) the boards of directors have the role of monitoring and advising, (2) the impact of a CEO's quality increases with the size of a firm under his control, (3) the CEO and the boards could be either complements or substitutes, and (4) the boards enjoy reputation value. When the reputation depends on the market value of firms, potential directors like to work at firms with talented CEOs. In contrast, when potential directors want value-added for reputation, they would be at firms with low-ability CEOs if the CEO and the boards are substitutes. Also, this model suggests the possibility that the quality of directors on the same boards could be dispersed, which is consistent with the empirical findings. My empirical estimates suggest that talented ongoing CEOs and former CEOs work as outside directors of firms with high market capitalization, though neither with high assets nor sales. The quality of boards is higher where the CEO pay is higher. I also find that the firms with high sales pay more to outside directors. A 1% increase in sales makes board compensation increase by 0.66%. Finally, board pay is 0.13% higher where CEO pay is 1% higher. We can infer that the CEO and the boards are complements.

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1 Introduction

Much research has explored the characteristics of boards and reached the consensus that efficient boards should be composed of a majority of outside directors. Colley and Stettinius (2003) argue that one of positive attributes of the boards of directors is to have no more than two inside directors on boards. The reason is that the boards satisfying this criterion are considered more independent. Since many people believe that outside directors would be more likely to represent shareholders' preferences due to relative independence, this view is widely accepted. It naturally follows that economic literature analyzes optimal board independence. Boards have two major functions: monitoring and advising on management. We can predict that these two functions are key determinants of board structures. Adams and Ferreira (2007) and Raheja (2005) provide theoretical background for the structure of boards. They argue that the board structure optimally responds to the benefit and cost of monitoring and advising. Lehn, Patro and Zhao (2004) and Linck, Netter and Yang (2006a) empirically support this argument.

There exists another important aspect of boards: the quality of boards. Fama (1980) argues that managers of high-performing firms are more likely to become outside directors of firms because the market values directors according to their efficacy as managers. The market thinks that they have the expertise to guide and monitor managerial behavior. Research also concludes that firms prefer highly qualified directors. (See Fama and Jensen (1983), Gilson (1990), Kaplan and Reishus (1990), Li (1997), Ferris, Jagannathan and Pritchard (2003) Keys and Li (2004), and Lee (2007c)). They, however, have studied the demand side only in the market for corporate directors and little light has been shed on the determination of the boards quality.

This paper develops a simple matching model in the director market with outside options to explain the equilibrium board quality. In line with the literatures studying the optimal board independence, the quality of outside directors on boards is a direct (advising) and an indirect (monitoring) mechanism that affects the earnings of a firm. An outside director contributes to the earnings of a firm directly by advising the CEO. Here, I consider two possible relationships between the CEO quality and the board quality: complement and substitute. While they can interact with each other positively, it could be the case that the CEO can dominate decision-making or the decisions of a CEO can be overridden by the board. The idea stems from Garicano and Rossi-Hansberg (2006). They propose a model to explain the organization of work in a knowledge economy and show a positive sorting in a sense that highly-qualified managers are matched with the higher-ability subordinates. The main driving force is that managers can share their ability with the workers under their control. Also, Garicano and Hubbard (2007) show the equilibrium assignment pattern in the non-hierarchical production function. The matching patterns mainly depend on the interaction of agents in the production of a firm. Second, a highly-qualified director increases the probability of finding the true quality of an incumbent CEO, which is defined by the monitoring. In addition, my model assumes

that the impact of CEO's quality increases in the size of a firm under his control, which is originally introduced by Gabaix and Landier (2008).[1] They propose a simple competitive assignment model in the CEO market to explain CEO compensation. They assume that CEOs have heterogeneous talent levels and are assigned to firms competitively. Also, the managerial impact of a CEO's talent increases with the value of the firm under his control. Under these assumptions, they suggest that the best CEO goes to the largest firms because the largest firm pays the most to the best CEO. Their empirical findings support these predictions. The CEO's pay increases with the size of a firm and the size of an average firm in the economy. Put simply, the most important determinant of the CEO compensation is the size of a firm, especially market capitalization. The reason for incorporating this assumption into my model is that the role of boards is to monitor and to advise the CEO, so the assignment pattern in the director market could also depend on the size of a firm, which is similar as the CEO market. Put simply, if the quality of CEO matters more at bigger firms, the quality of boards could be more valuable there.

On the demand side of the market for corporate directors, the firm would like to fill a vacancy with a candidate satisfying the minimum quality level. The minimum quality level is analyzed based on Pissarides (2000). He provides excellent work for searching and matching in the labor market to incorporate the market condition (matching function). On the supply side, a potential candidate for outside directors compares the reputation value generated by the directorship[2] to the outside option value.[3] The reputation value could depend on either the market value of a firm or the market value added by a potential director.[4] If the outside option is good for the highly qualified candidates, they would not contact the firm which creates a vacancy for the outside director. Henceforth, the quality of a new director is determined by random matching between the minimum quality level required by a firm and the cutoff quality level of the potential candidates who are likely to contact the firm. More importantly, the main driving force to determine the equilibrium quality level is the cutoff quality level. When the firms face the high cutoff quality level, they can set up a high minimum quality level because they would be more likely to meet the more talented candidates in the future. In other words, the assignment pattern in the director market is mainly determined by scarcity of the talented candidates. The

[1] We can interpret this assumption as follows: (1) The real power comes from the amount of resources which the CEO can allocate. (2) The "Size-Skill Complementarities" exists in the hierarchy of a firm. Garicano and Rossi-Hansberg (2006) suggest that the ability of managers could be amplified by the amount of controllable resources in the hierarchy of firms because talented employees can share their ability (or knowledge) with the team under their control. In what follows, the more talented employees hold higher positions in the equilibrium.

[2] Hambrick and Johnson (2000) said "The majority of outside directors are fully motivated to act conscientiously and vigorously by forces other than a financial stake in the firm: their sense of professionalism, concern for their reputations and stature, and the threat of lawsuit." (Colley and Stettinius (2003), page 61). Additionally, the empircal evidence in this paper suggests that there is no correlation between the quality of boards and the pay for directors.

[3] We can easily extend this model to the case that a potential director cares only money value (pay for directors) or both money value and reputation value assuming that the pay for director is determined by the marginal contribution of boards to the market value of a firm.

[4] Eric Rasmusen suggests this possibility for reputation value. A potential director would be likely to want influence, importance, value-added, for its own sake.

qualified candidates for outside directors, specifically ongoing and retired CEOs, have high demand in the professional labor market. In contrast, when the firms would like to fill a vacancy on boards, they face scarcity in the market for talent.

This model suggests the possible matching pattern between firms and potential directors. In general, the potential directors like to work at big firms. They could give greater reputation values. The matching pattern between the firm with high or low ability CEO and potential director mainly depends on (1) the interaction of both the agents, and (2) the definition of reputation values. Suppose the reputation value only depends on the market value of a firm. Whatever the relationship is "complement" or "substitute", the potential candidates like to work at firms with talented CEOs. However, when the reputation is generated by the market value added by potential directors, they would be at firms with low-ability CEO's if the CEO and the boards are substitutes. They feel more happy when they help lower-ability CEO's. Also, this model suggests the possibility that the quality of outside directors on the same boards could be very dispersed, which is the random matching between the minimum quality level and the cutoff (maximum) quality level, but the highest quality, the lowest quality, and the average quality on boards would be systematically correlated with the size of a firm and the quality of a CEO. These predictions are consistent with the empirical findings in this paper.

To measure the quality of boards, I focus on the CEO's working as outside directors. The popular outside directors in U.S. large firms are the CEO's. My data shows that almost half ($\approx 47\%$) of outside directors are ongoing or retired CEOs. I use the CEO pay and market capitalization of firms at which ongoing/retired CEO's are working/worked as a proxy for the quality of directors. Suppose A outside director on boards of Wall Mart is an ex-CEO of GM. The quality of A is measured by the size (market capitalization) of GM or by the CEO compensation GM paid to A. Gabaix and Landier (2008) argue that the best CEO goes to the largest firm (positive assortative matching) in a competitive assignment market because the largest firm pays the most to the best CEO. Their empirical finding supports this argument. The firm size (specifically, market capitalization) is the most important and observable determinant of CEO pay. In this sense, the CEO pay or the size of a firm could be a good proxy for the quality of ongoing or retired CEO. Also, Lee (2007c) analyzes directorships held by 250 CEOs who retired during 1998-2002 in the two years after retirement. He finds that the firm size (total assets) in which the CEO worked before retirement is directly related to the number of outside directorships.[5] Firms prefer retired CEOs from large firms as outside board members.[6]

[5] Brickley, Linck and Coles (1999) also show that the firm size in which CEOs worked before retirement explains well the number of outside directorship held by CEOs 2 years after retirement.

[6] Additionally, the accounting performance is a possible candidate. Brickley, Linck and Coles (1999) analyze directorships held by 277 CEOs who retired during 1989-1993 after they retired and show that the accounting performance (ROA and industry adjusted ROA) of CEOs during the final 4 years in office has an economically significant effect on the number of outside board seats they hold after retirement. Lee (2007c), however, finds that the accounting performance is not significantly correlated with the number of outside directorships in the 1998-2002 sample. There are some possibilities which could explain this change; (1) Accounting performance might lose the credibility as the index for the performance due to

There are some stylized facts in the market for corporate directors. The average quality of boards is not significantly different with the size of a firm.[7] Put differently, the average director of GE is from the similar-sized firm. However, the quality of directors on the same boards is very dispersed. I rank ongoing CEOs and former CEOs on boards of U.S. large firms in 2005 by the compensation levels as CEOs. Then, I measure the dispersion of quality by the difference in ranking between the best director and the worst director on the same boards. The mean value is 226.88, which means that if the best director of Intel is the first-ranked one in the pool of outside directors, the ranking of the worst director on Intel boards is approximately 228th. Also, the best director of firm j tends to be the CEO of bigger firm than firm j, but the lowest-ability director is from the smaller firm than firm j.

My estimates suggest that talented CEO's and former CEO's work as directors of firms with high market capitalization. This implies that market capitalization matters more as an indicator of firm size (value) than total assets and sales. Second, the quality of boards is higher where the CEO pay is higher. Third, board quality is not correlated with board compensation. In addition, I analyze the board compensations. I find that the firms with high sales pay more to outside directors. When the amount of sales increases by 1%, the board compensation (annual director fee) increases by 0.66%. Also, when the CEO compensation increases by 1%, the board compensation increases by 0.13%. We can infer that the CEO and the boards are complements.

The rest of the paper is organized as follows. Section 2 provides a brief review of the related literature. In section 3, I develop a model and provide the empirical predictions. Section 4 describes the data set and the empirical results. I summarize concluding remarks in Section 5.

Enron scandals or (2) Firms increasingly put more weight on general skills rather than firm-specific skills when they select board members. One of Brickley, Linck and Coles (1999)'s potential explanations for the strong relationship between the accounting returns and the number of outside directorships is that the accounting performance might reflect a talented CEO's ability under the condition that the CEO's skills are specific to the firm because the firm could share the difference between CEO's value to the firm and his next highest-value. In this sense, if the firm increasingly requires general skills of outside board members, the accounting performance might be less important in the selection of outside directors. The stock return during tenure is another possibility, but the stock return of the old company is not good enough. If a CEO is predicted when he begins his job to be good, then his company should just have a normal market return, not above-market. An above-market return only indicates he is doing better than expected."

[7] The average quality of boards is calculated as follows; Suppose that there are three outside directors (A, B, and C) on boards of GE. Then,

$$\text{the average quality of GE boards} = \frac{\text{the size of A's own firm+B's own firm+C's own firm}}{3}$$

I test the equality of matched pairs of observations (the average quality of boards and the size of a firm) by using "signtest" and "signrank" command in Stata.

2 Related literature

This paper is mainly related with a field which studies the quality of boards. It focuses on the relationship between the quality of potential candidates for outside directors and the probability of serving as outside directors on boards.

2.1 The quality of boards

Fama (1980) argues that managers of high-performing firms are more likely to become outside directors of firms because the market prices directors according to their efficacy as managers. The market thinks that they have attributes necessary to guide and monitor managerial behavior. Kaplan and Reishus (1990) find that the probability of a CEO taking on an outside directorship is positively related to their firm's performance. Li (1997) provides evidence that the labor market for directors is well functioning and the market prices directors based on their performance as directors. Brickley, Linck and Coles (1999) analyze directorships held by 277 CEOs who retired during 1989-1993 after they retired and show that accounting performance (ROA and industry adjusted ROA) of CEOs during the final 4 years in office has an economically significant effect on the number of outside board seats they serve on after retirement. Market performance, however, does not explain it. Ferris, Jagannathan and Pritchard (2003) find that the performance of a firm on which he has served as a director has a positive effect on the number of other directorships (other firms' outside directorships) held by him, which is consistent with Fama and Jensen (1983). Keys and Li (2004) find that professional directors are three times more likely to receive additional directorships following a successful tender offer for a firm on which they served as board members. Lee (2007c) analyzes directorships held by 250 CEOs who retired during 1998-2002 in the two years after retirement and finds counter-evidence that pre-retirement accounting performances cannot explain the number of outside directorships held by CEOs two years after retirement.

Overall, much research about corporate governance focuses on the relationship between the quality of candidates for outside directors and the probability of serving as directors. This research concludes that the number of directorships a candidate holds increases with his quality level. However, little light has been shed on the equilibrium in the market for corporate directors.

3 Model

3.1 Model: Searching and Matching

I construct a searching and matching model in which both potential candidates for outside directorships and firms live forever and are risk neutral. The basic framework stems from Pissarides (2000). There are potential candidates for outside directorships and firms which are normalized to 1. Every potential candidate i with heterogeneous quality, q^i_{new},

has an outside option,[8] so that their choice is whether to serve as an outside director or enjoy an outside option. The choice of the firm is to decide whether to fill a vacancy for an outside directorship now or to wait a better matching in the next period. Neither quitting nor firing are allowed. The quality, q^i_{new}, is $\in [0, 1]$.

The vacancies and potential candidates who would be likely to contact (a subgroup of potential candidates) are assumed to meet each other randomly in the director market. When a vacancy is created, the firm j determines the minimum required quality level and post it. Then, the firm j directs its search effort toward the potential candidates satisfing this level. Also, the potential candidates who can enjoy the higher value to work as an outside director in the firm j than the outside option value would be likely to contact the firm j. The talented candidates are assumed to have good outside options, so that the candidates below the cutoff quality level stay and search in the market for corporate directors. The firm j meets potential candidates who would be likely to contact at the rate ϑ_j.[9] I will explore below the nature of steady-state equilibrium and focus on the quality level of outside directors on boards. Hereafter, I omit the subscript i and j.

3.2 The value function

First, I consider the supply side to develop the value function of a potential candidate for outside directorships. The problem of a potential candidate with the quality q_{new} is

$$V = \max \{V_c(q_{new}), V^o_c(q_{new})\}$$

where $V_c(q_{new})$ and $V^o_c(q_{new})$ represent the value of an outside directorship and an outside option. I begin with the value of an outside directorship, $V_c(q_{new})$.

$$V_c(q_{new}) = \underbrace{E[\pi(q_{new})]}_{\text{the reputation value}} + \frac{1}{1+\gamma} V_c(q_{new}),\ q^{\min}_{new} \leq q_{new} \leq q^{cut}_{new} \tag{1}$$

The value of an outside directorship equals the reputation value generated by the outside directorship plus $\frac{1}{1+\gamma} V_c(q_{new})$. The reputation value is defined by $E[\pi(q_{new})]$, where $E[\pi(q_{new})]$ is the expected market value of a firm when a potential candidate with quality, q_{new}, joins on boards.[10] γ represents the discount rate. Finally, q^{cut}_{new} and $q^{\min}_{new}$ are the cutoff quality level and the required minimum quality level for a new director. We can

[8] This assumption captures the following facts. Many CEOs have several job position opportunities after retirement except outside directorship (community board, government organization, officer in private firms, consultant and so on). Ongoing CEOs also have many similar options. Also, ongoing CEOs usually face the time allocation problem.

[9] In the classical random searching model, the contacting (meeting) function is given by

$$m(u, v) = m(\frac{v}{u})u = m(\theta)u,\ \theta = \frac{v}{u}$$

where u is the unemployment rate and v is the measure of vacancies. Under the assumptions that all workers are the same and all firms are same, vacancies meet unemployed workers at the rate $\frac{m(\theta)}{\theta}$.

[10] The market value of a firm actually depends on the average quality level of outside directors on boards with a new director. For the sake of exposition, I express the value function in terms of q_{new}. I will go in details later.

rewrite the equation (1) by

$$V_c(q_{new}) = \frac{(1+\gamma)E[\pi(q_{new})}{\gamma} \tag{2}$$

The outside option value, $V_c^o(q_{new})$, is defined by

$$V_c^o(q_{new}) = y(q_{new}) + \frac{1}{1+\gamma}V_c^o(q_{new}), q_{new} \geq q_{new}^{cut} \tag{3}$$

The outside option value equals the return of an outside option plus $\frac{1}{1+\gamma}V_c^o(q_{new})$. The return of an outside option is assumed to have a functional form denoted by $y(q_{new})$, $y'(q_{new}) > 0$. We can rewrite the equation (3) by

$$V_c^o(q_{new}) = \frac{(1+\gamma)y(q_{new})}{\gamma} \tag{4}$$

Second**,** I consider the demand side, the vacancy for an outside director. Each firm faces a decision whether to fill a vacancy for an outside director now or to wait a better matching in the next period. The problem of a firm is

$$V = \max\left\{V_f(q_{new}), V_f^s)\right\}$$

where $V_f(q_{new})$ and $V_f^s(\overline{q_s})$ denote the value of filling a vacancy and the value of not filling a vacancy. The value of filling a vacancy is given by

$$V_f(q_{new}) = E[\pi(q_{new})] + \frac{1}{1+\gamma}V_f(q_{new}),\ q_{new}^{\min} \leq q_{new} \leq q_{new}^{cut}$$

The return of filling a vacancy is the expected market value of a firm with a new director. Then, we can rewrite the value of filling a vacancy by

$$V_f(q_{new}) = \frac{(1+\gamma)E[\pi(q_{new})]}{\gamma} \tag{5}$$

The value of not filling a vacancy is given by

$$V_f^s = E[\pi(\overline{q_s})] + \frac{1}{1+\gamma}\left\{\vartheta E V_f(q'_{new}) + (1-\vartheta)V_f^s(\overline{q_s})\right\},\ q_{new} < q_{new}^{\min} \tag{6}$$

where ϑ[11] denotes the arrival rate of potential candidates who try to find an outside director position, $EV_f^{new}(q'_{new})$ is the expected value of filling a vacancy with a candidate having the quality level, q'_{new}, in the next period. $\overline{q_s}$ represents the average quality level of boards when the firm does not fill a vacancy. Rearranging the equation (6), we get

$$V_f^s = \frac{(1+\gamma)\left(E[\pi(\overline{q_s})] + \vartheta E V_f^{new}(q'_{new})\right)}{\gamma+\vartheta}$$

[11]Note that ϑ is endogenously determined. I will discuss about this later on.

3.3 The market value of a firm

Here, I will derive the expected market value of a firm. Based on Hermalin and Weisbach (1998), Holmstrom (1999) and Hermalin (2005), I set up the timing for the following. At the first stage, the firm and potential candidate for outside directorships only have the prior distribution for the quality of an incumbent CEO, q^{μ}, [12] which has a mean $\mu > 0$. In the second stage, the firm and potential candidate (if he joins on boards) would be likely to find the true quality of the CEO, q^{μ} with the probability equal to the average board quality. Otherwise, the firm and potential candidate learn nothing. If the firm chooses a new outside director with the quality level, q_{new}, then the firm can find the true quality with probability equal to the average board quality given by

$$\overline{q_n} = \frac{(n-1)\overline{q_s} + q_{new}}{n}, \tag{7}$$

but the firm learns nothing about the incumbent CEO with probability $1 - \overline{q_n}$. n is the total number of outside directors including a new outside director. Similarly, if the firm does not fill a vacancy and wait for a better matching in the future, the firm can find the true quality with probability equal to the average quality of boards without a new outside director, $\overline{q_s}$, but the firm learns nothing about the incumbent CEO with probability $1 - \overline{q_s}$. Finally, the firm decides whether to fire the incumbent CEO or not based on the true quality q^u or the prior expectation of the quality of the CEO, and then the market value of a firm is realized. If the incumbent CEO is fired, the market value of a firm depends on the quality of a replaced CEO. The quality of a potentially replaced CEO is randomly distributed with mean $0 < \mu$.[13] If the firm and potential candidate find the true quality of an incumbent CEO, the incumbent CEO is fired when

$$q^{\mu} < 0$$

When the firm finds nothing, the incumbent CEO is retained because the firm decides whether to fire the incumbent CEO or not based on the prior expectation, $\mu > 0$. This three-stage process is iterated at each period because the quality of a CEO is specific to the project implemented at each period and both the firm and potential candidate are uncertain about it.

3.3.1 CEO quality and Board quality

Case 1: Complement Now, I will derive the expected market value of a firm which depends on the quality of an incumbent, a replaced CEO and outside directors. I assume the symmetric complementarities (Becker (1981) and Becker (1993)) between the

[12] This assumption implies that it is uncertain that the CEO's (general) skill would be well matched with the firm-specific project and environment.

[13] This assumption guarantees that an incumbent CEO will not be fired when the firm finds nothing about the incumbent CEO.

CEO quality and board quality.[14] Based on Murphy and Zabojnik (2004) and Gabaix and Landier (2008), the expected market value of a firm when a vacancy is filled by a new outside director, $E[\pi(q^k, \overline{q_n})]$, is given by

$$E[\pi(q^k, \overline{q_n})] = \underbrace{S^\sigma E[q^k]\overline{q_n}}_{earning} - \underbrace{W_{CEO} - nW_{BOARD}}_{cost}, k = u \text{ or } r,\ 0 < \sigma < 1 \qquad (8)$$

where S denotes the size of a firm, [15] q^u is the quality of an incumbent CEO, q^r is the quality of a replaced CEO, and $\overline{q_n}$ denotes the average quality of boards with a new outside director. σ is "return to scale" parameter. Also, W_{CEO} and W_{BOARD} represent the compensation of CEO and boards. n is the total number of outside directors including a new outside director. Similarly, the expected market value of a firm when a vacancy is not filled, $E[\pi(q^k, \overline{q_s})]$, is given by

$$E[\pi(q^k, \overline{q_s})] = S^\sigma E[q^k]\overline{q_s} - W_{CEO} - (n-1)W_{board}$$

where $\overline{q_s}$ is the average quality of boards without a new outside director. The expected market value of a firm with a new outside director (the equation (8)) is rewritten by

$$\begin{aligned} E[\pi(q^k, \overline{q_n})] \quad = \quad & \underbrace{\overline{q_n} S^\sigma F(q^u) q^u \overline{q_n}}_{\text{the expected earning with the true quality}} + \underbrace{(1-\overline{q_n}) S^\sigma \mu \overline{q_n}}_{\text{the expected earning with nothing}} \\ & - W_{CEO} - nW_{board} \end{aligned}$$

where the firm finds the true quality with the probability $\overline{q_n}$ and nothing with the probability $1 - \overline{q_n}$. $F(q^u)$ is the retaining probability of an incumbent CEO. We can easily find $E[\pi(q^k, \overline{q_s})]$ by the similar method. Finally, we can get the expected market value of a firm with (or without) a new director, $E[\pi(q^k, \overline{q_i})]$ by [16]

$$E[\pi(q^k, \overline{q_i})] = \Omega \overline{q_i}^2 + S^\sigma \mu \overline{q_i} - W_{CEO} - n(\text{or } n-1)W_{BOARD}, i = n \text{ or } s \qquad (9)$$

where

$$\Omega(\mu, S) = S^\sigma \left[F(q^u) q^u - \mu\right]$$

Case 2: Substitute Let me suppose that the CEO and outside directors are substitutes.[17] By adopting the substitute form originally introduced by Sah and Stiglitz

[14] Alternatively, we can consider the production function which is asymmetrically sensitive to the quality of CEO and of boards, for instance,

$$S^\sigma (E[q^k])^\theta (\overline{q_n})^{1-\theta}$$

This is originally proposed by Kremer and Maskin (1996). This, however, does not affect the qualitative prediction of model in this paper.

[15] This set-up reflects the "size-skill Complementarity". See Gabaix and Landier (2008) for more details.

[16] You can find the similar setting in Hermalin (2005).

[17] Eric Rasmusen points out that CEO quality and board quality can be substitutes in a sense that the good CEO can dominate the decision-making or the decision of the CEO can be overriden by the good boards.

(1986), the expected market value of a firm with (or without) a new director, $E[\pi(q^k, \overline{q_i})]$, is given by

$$\begin{aligned} E[\pi(q^k, \overline{q_i})] &= \underbrace{\overline{q_i} F(q^u) S^{\sigma} \left[a - (q^u_{\max} - q^u)(1 - \overline{q_i})\right]}_{\text{the expected earning with the true quality}} \\ &\quad + \underbrace{(1 - \overline{q_i}) S^{\sigma} \left[c - (u_{\max} - u)(1 - \overline{q_i})\right]}_{\text{the expected earning with nothing}} \\ &\quad - W_{CEO} - n(\text{or } n-1) W_{BOARD} \end{aligned} \tag{10}$$

where a and c are given constants. $q^u_{\max}$ and $u_{\max}$ represent the maximum value for the true quality of an incumbent CEO and the maximum value of the prior expectation of the incumbent CEO's quality, respectively.

3.4 The cutoff quality level

As I mentioned above, the quality of a new director is determined by random matching between the minimum quality level and the cutoff quality level. Now, I will solve the endogenous cutoff quality level, q^{cut}_{new}, to incorporate the expected market value of a firm. Putting the equation (2) and (4) together, we get the following equation which characterizes the cutoff level

$$E[\pi(q^k_c, q^{cut}_{new})] = y(q^{cut}_{new}) \tag{11}$$

The left side represents the benefit of holding an outside directorship, and the right side is the benefit of an outside option. The cutoff level is determined to equate the equation (15). Hereafter, I suppress the cost part of $E[\pi(q^k_c, q^{cut}_{new})]$ in the equation (9) for the simplicity.

Proposition 1 *Under some assumptions, there exist a unique cutoff quality level which guarantees that the potential candidates with $q_{new} \leq q^{cut}_{new}$ are likely to contact. Then, (1) the cutoff quality level increases in the size of a firm in both "complement" and "substitute" case. (2) The cutoff quality level increases in the prior expectation for the quality of an incumbent CEO (hereafter, the CEO quality) in both "complement" and "substitute" case.*

Proof. See in Appendix ■

3.5 The minimum quality level

Putting the equation (5) and (6) together, we can get the following equation to characterize the minimum quality level.[18]

$$E[\pi(q_c^k, q_{new}^{\min})] = \gamma \left\{ \frac{(1+\gamma)\left(E[\pi(q_c^k, \overline{q_s})] + \vartheta EV_f^{new}(q'_{new})\right)}{\gamma + \vartheta} \right\} \tag{12}$$

The left-hand side represents the cost and the right-hand side the benefit of one more searching. Now, I will define the functional form of the arrival rate. For simplicity, suppose that each candidate in $q_{new}^{\min} \leq q'_{new} \leq q_{new}^{cut}$ is equally acceptable to a subset of firms which has created a vacancy. Then, the arrival rate of a potential candidate with quality, q'_{new} in $q_{new}^{\min} \leq q'_{new} \leq q_{new}^{cut}$ to the firm can be assumed to be given by

$$\vartheta(q'_{new}) = \psi h(q'_{new})^{19}$$

where ψ captures the property that each candidate in $q_{new}^{\min} \leq q'_{new} \leq q_{new}^{cut}$ is equally acceptable.[20] $h(q'_{new})$ represents the density of a potential candidate with quality, q'_{new}. Then the aggregate arrival rate which satisfies $q_{new}^{\min} \leq q'_{new} \leq q_{new}^{cut}$ is given by

$$\vartheta = \int_{q_{new}^{\min}}^{q_{new}^{cut}} \psi h(q'_{new}) dq'_{new}, \ \frac{\partial \vartheta}{\partial q_{new}^{cut}} > 0, \ \frac{\partial \vartheta}{\partial q_{new}^{\min}} < 0 \tag{13}$$

Plugging equation (13) into (12), we can get

$$E[\pi(q_c^k, q_{new}^{\min})] \tag{14}$$

$$= \gamma \left\{ \frac{(1+\gamma)E[\pi(q_c^k, \overline{q_s})] + \left(\int_{q_{new}^{\min}}^{q_{new}^{cut}} \psi h(q'_{new}) dq'_{new}\right)\left(\frac{(1+\gamma)E[\pi(q'_{new})]}{\gamma}\right)}{\gamma + \int_{q_{new}^{\min}}^{q_{new}^{cut}} \psi h(q'_{new}) dq'_{new}} \right\}$$

Proposition 2 *Under some assumptions, there exists a unique quality level which guarantees that the firm would like to fill a vacancy with a potential candidate satisfying* $q_{new}^{\min} \leq q_{new}$. *Also, (1) the minimum quality level increases in the size of a firm if the effect of the firm size on the cutoff quality level,* q_{new}^{cut}, *is large enough (2) The minimum quality level increases in CEO quality if the effect of the CEO quality on the cutoff quality level,* q_{new}^{cut}, *is large enough.*

Proof. Omitted ■

[18] See Ljungqvist and Sargent (2004), page 88.

[19] We can think of the arrival rate as a simple functional form of $\sigma \frac{u}{v}$ (σ is constant), where u is the unemployed rate and v is the measure of vacancy. ψ can be interpreted as $\frac{1}{v}$.

[20] If each candidate in $q_{new}^{\min} \leq q'_{new} \leq q_{new}^{cut}$ is unequally acceptable to a subset of firms which create a vacancy, ψ would be the function of q'_{new}. For instance, if highly qualified candidates are acceptable to a large subset of firms, ψ is decreasing in q'_{new}. If each candidate in $q_{new}^{\min} \leq q'_{new} \leq q_{new}^{cut}$ is equally acceptable, ψ can be assumed to be invariant across q'_{new}.

Both the benefit and the cost of one more searching becomes higher at bigger firms, so, if the effect of the firm size on the cutoff quality level is large enough, the marginal benefit could outweigh the marginal cost of one more searching. It naturally follows that the minimum quality level increases in the size of a firm. When the cutoff level rises, the aggregate arrival rate ϑ goes up, which implies that the firms are more likely to meet talented candidates in the future. Henceforth, the firms could set up the higher minimum quality level now. This logic could be applied to the rise in the CEO quality, too.

3.6 The steady-state equilibrium quality of boards

The matching is randomly consummated between the potential candidate in $q_{new}^{\min} \leq q_{new} \leq q_{new}^{cut}$ and the firm requiring $q_{new}^{\min}$ because all matches satisfy the following conditions:

$$V_c(q_{new}) + V_f^{new}(q_{new}) > V_c^o(q_{new}) + V_f^s(q_{new}),\ q_{new}^{\min} \leq q_{new} \leq q_{new}^{cut}$$

The equilibrium quality of a new director q_{new}^* is defined by

$$q_{new}^* = \left(\int_{q_{new}^{\min}}^{q_{new}^{cut}} \Phi^{\Xi}(q_{new}) q_{new} dq_{new} \right)$$

where $\Phi^{\Xi}(q_{new})$ represents the de-generated probability that a potential candidate with q_{new} in $\Xi = \{q_{new} \mid q_{new}^{\min} \leq q_{new} \leq q_{new}^{cut}\}$ first meets the firm. For the sake of exposition, the de-generated probability that a potential candidate with q_{new} in $\Xi = \{q_{new} \mid q_{new}^{\min} \leq q_{new} \leq q_{new}^{cut}\}$ first meets the firm is assumed to have the functional form of

$$\Phi^{\Xi}(q_{new}) = \left(\frac{1}{q_{new}^{cut} - q_{new}^{\min}} \right)$$

Then, the equilibrium quality of a new director q_{new}^* boils down to

$$q_{new}^* = \frac{q_{new}^{cut} + q_{new}^{\min}}{2} \tag{15}$$

Proposition 3 *(1) The equilibrium quality of a new director q_{new}^* would increase in the size of a firm, S if the effect of firm size on the cutoff quality level is large enough (2) The equilibrium quality of a new director q_{new}^* would increase in CEO quality if the effect of CEO quality on the cutoff quality level, q_{new}^{cut}, is large enough.*

Proof. Omitted ■

The equilibrium quality of a new director q_{new}^*, is determined by the random matching between $q_{new}^{\min}$ and q_{new}^{cut} as shown in equation (15), so the channel through which the size of a firm and CEO quality have effects on the equilibrium quality of a new director is straightforward.

3.7 Reputation: "Value added by me"

Here, I assume that the reputation value of a potential director depends on the expected market value added by a potential director, not the expected market value of a firm. Then, the value of an outside directorship is given by

$$V_c(q_{new}) = \underbrace{E[\frac{\partial \pi(q_{new})}{\partial q_{new}}]}_{\text{the reputation value}} + \frac{1}{1+\gamma} V_c(q_{new}),\ q_{new} \geq q_{new}^{\min}$$

where $E[\frac{\partial \pi(q_{new})}{\partial q_{new}}]$ represents the market value of a firm added by a potential candidate with quality, q_{new}.

Proposition 4 *Suppose that the CEO and the boards are substitutes. Then, (1) the equilibrium quality of a new director q_{new}^* would increase in the size of a firm if the effect of firm size on the cutoff quality level is large enough, and (2) the equilibrium quality of a new director q_{new}^* would decrease in CEO quality.*

Proof. Omitted ■

When the CEO is expected to be good, the influence of boards (the marginal contribution of a director to the market value of a firm) becomes weaker because the better CEO is more likely to dominate the decision-making of a firm. It follows that the cutoff quality level and the minimum quality level decline.

3.8 The implications for the board compensations

From the expected market value of a firm we derived before, we can find some implications for the pay for directors.

Case 1: Complement The compensation to each board member is assumed to be the marginal contribution of the average board quality to the earning of a firm divided by the number of board members, which is given by

$$W_{BOARD} = \frac{2\Omega(\mu, S)\overline{q_i} + S^{\sigma}\mu}{n\ (or\ n-1)}, \tag{16}$$

The marginal contribution of the average board quality is composed of the marginal contribution of monitoring and advising. The marginal contribution of monitoring is expressed by $\Omega(\mu, S)\overline{q_i}$, which represents the expected earning from the perfect information for the incumbent CEO minus from the prior information. The marginal contribution of advising is given by $\Omega(\mu, S)\overline{q_i} + S^{\sigma}\mu > 0$.

Proposition 5 *(1) The compensation for directors increases in the size of a firm. (2) The compensation for directors increases in the prior expectation for the quality of an incumbent CEO (hereafter, the CEO quality) if the average board quality is less than 1/2.*

Proof. See in Appendix ■

(1) The firms being big make the role of monitoring and advising more valuable. The CEO matters more at bigger firms, so that the role of monitoring and advising CEO also becomes more valuable. (2) When the CEO is expected to be good, there is a "trade-off" between monitoring and advising. Since, from the viewpoint of firms, there is little incentive to find perfect information for the quality of CEO, the monitoring role of boards becomes less important. The advising role, however, becomes more important owing to "complementarity" between CEO and boards. The impact of good CEO to the earning of a firm could be amplified by hiring good directors.

Case 2: Substitute In this case, the compensation given to each board member is

$$W_{BOARD} = \frac{S^{\sigma}\left(F(q^u)\left[a-(q^u_{\max}-q^u)(1-\overline{q_i})\right]-\left[c-(u_{\max}-u)(1-\overline{q_i})\right]\right)}{n\ (or\ n-1)} + \frac{S^{\sigma}\overline{q_i}F(q^u)(q^u_{\max}-q^u)+S^{\sigma}(1-\overline{q_i})(u_{\max}-u)}{n\ (or\ n-1)} \quad (17)$$

Proposition 6 *(1) The compensation for directors increases in the size of a firm. (2) The compensation for directors decreases in the CEO quality.*

Proof. Omitted ■

The reason for (1) is the same as the proposition 1. (2) Be careful that the advising role becomes less important when the CEO is expected to be good. The better CEO could make the boards less important even in terms of advising to dominate the decision-making of a firm.

4 Data and empirical result

4.1 Proxy for quality of boards

Board members are usually composed of a firm's executives (CEO, CFO etc.,), other firms' CEOs, executives, retired CEOs, lawyers, professors, and so on. In this paper, I focus on the quality of outside directors, especially ongoing CEOs and retired CEOs from the outside. When we proxy the quality of retired CEOs and current CEOs, there are two possible candidates. The first one is the total compensation paid to them when they worked/are working as CEOs.[21] Second, the firm size at which they worked before retirements/are working is a good proxy for the quality. For instance, A outside director

[21] In a slightly different angle, Garicano and Rossi-Hansberg (2006) show that the equilibrium wage is increasing and convex in the ability of agents in the hierarchies of firms because the top managers share their ability with a team under their control. The firms pay top managers more than proportional to their talent. So, the log value of the wage could be the better proxy for the quality of ongoing/retired CEOs.

on boards of Wall Mart is an ex-CEO of GM. The quality of A is measured by the CEO compensation GM paid to A or by the size (market capitalization) of GM. This idea stems from Gabaix and Landier (2008), Brickley, Linck and Coles (1999), and Lee (2007c). Gabaix and Landier (2008) develop the model which shows that the best CEO goes to the largest firm (positive assortative matching) in a competitive assignment market because the largest firm pays the most to the best CEO. Their empirical finding supports this argument. The firm size (specifically, market capitalization) is the most important and observable determinant of CEO compensations. Brickley, Linck and Coles (1999) show that the firm size in which CEOs worked before retirement explains well the number of outside directorship held by CEOs 2 years after retirement during 1989-1993. Lee (2007c) also analyzes directorships held by 250 CEOs who retired during 1998-2002 in the two years after retirement. He finds that the firm size (total assets) in which the CEO worked before retirement is directly related to the number of outside directorships. Firms prefer retired CEOs from large firms as outside board members. Additionally, the accounting performance is a possible candidate. Brickley, Linck and Coles (1999) analyze directorships held by 277 CEOs who retired during 1989-1993 after they retired and show that the accounting performance (ROA and industry adjusted ROA) of CEOs during the final 4 years in office has an economically significant effect on the number of outside board seats they hold after retirement. Lee (2007c), however, finds that the accounting performance is not significantly correlated with the number of outside directorships in the 1998-2002 sample. There are some possibilities which could explain this change; (1) Accounting performance might lose the credibility as the index for the performance due to Enron scandals or (2) Firms increasingly put more weight on general skills rather than firm-specific skills when they select board members. One of Brickley, Linck and Coles (1999)'s potential explanations for the strong relationship between the accounting returns and the number of outside directorships is that the accounting performance might reflect a talented CEO's ability under the condition that the CEO's skills are specific to the firm because the firm could share the difference between CEO's value to the firm and his next highest-value. In this sense, if the firm increasingly requires general skills of outside board members, the accounting performance might be less important in the selection of outside directors. The stock return during tenure is another possibility, but the stock return of the old company is not good enough. If a CEO is predicted when he begins his job to be good, then his company should just have a normal market return, not above-market. An above-market return only indicates he is doing better than expected.

4.2 The average quality of boards: sample selection

I select 266 firms among *Fortune 500* U.S firms in 2005 and collect board profiles of those firms in the same year. I gather board information from the *Securities and Exchange Commission (SEC) filings* to search each firm's proxy statement (*filing form:*

DEF 14A).[22] First, I classify board members into insiders and outsiders by adopting the following classification method.[23]

$$Inside\ director\ =\ current\ employee + former\ employee$$

$$Outside\ director = \ All\ other\ directors$$

The outside directors are mainly composed of other firms' ongoing CEOs, retired CEOs, executives, lawyer and professors. *Table 1* shows descriptive statistics for our sample. Almost half of the outside directors on boards are ongoing CEOs in other firms plus retired CEOs from other companies. The boards tend to contain more retired CEOs (2.5) than ongoing CEOs (1.99).

I use the CEO compensation and the size of a firm (market capitalization[24]) as the proxy for the quality of boards. I find the executive compensation data for 487 ongoing or retired CEOs and the market capitalization value of 659 ongoing or retired CEOs on boards. In the case of ongoing CEOs, I use the compensation/market capitalization in 2004. For retired CEOs, I collect the compensation/market capitalization one year before retirement and then convert all values into 2004 year value, using the median growth rate of CEO compensation/market capitalization. The first observed fact is that the average quality of boards (measured by market capitalization) is not different with the size of a firm.[25] Put differently, the average director of GE is from the similar-sized firm. *Table 2* shows the descriptive statistics for the average quality of boards. Here, the proxies for the quality is the averaged CEO compensation levels.[26] The Wilcoxon rank-sum test shows that the quality of boards averaged over only retired CEOs on boards is significantly higher than the quality of boards averaged over only ongoing CEOs on boards (See *Table*

[22] The Investor Responsibility Research Center (IRRC) provides board and committee information. I, however, need more detailed information for board members, so I handy-collect profiles from *SEC* filings.

[23] This classification method is used in Linck, Netter and Yang (2006a). Lehn, Patro and Zhao (2004) classify directors into three categories: (1) inside directors, (2) outside directors, and (3) gray directors

[24] Following Gabaix and Landier (2008), the market capitalization is defined as the sum of market value of equity and book value of debt.

[25] The average quality of boards is calculated as follows; Suppose that there are three outside directors (A, B, and C) on boards of GE. Then,

$$\text{the average quality of GE boards}$$

$$= \quad \frac{\text{the size of A's own firm+B's own firm+C's own firm}}{3}$$

I test the equality of matched pairs of observations (the average quality of boards and the size of a firm) by using "signtest" and "signrank" command in Stata.

[26] The average quality of boards is measured as follows; Suppose there three outside direcotors on boards of Dell, A,B, and C. A is ongoing CEO of Coca-Cola, B is ex-CEO of Ford, and C is ex-CEO of Yahoo. Then,

$$\text{the average quality of Dell boards}$$

$$= \quad \frac{\text{the CEO pay of A at Coca-Cola+B at Ford+C at Yahoo}}{3}$$

2-A). *Table 2-B* also supports this finding. The mean quality of retired CEOs who work as outside directors in 2005 is significantly higher than the mean quality of ongoing CEOs on boards. The last column of *Table 2-A* provides evidence that the quality of directors on the same board is very dispersed. The mean level of the difference in ranking between the highest and the lowest on the same board is 226.88. For instance, the best director of Intel is the first-ranked one in the pool of outside directors, the ranking of the worst director on Intel boards is approximately 228th. There is another evidence for the dispersion of quality on the same boards. As I mentioned above, the average quality of boards measured by the market capitalization is not significantly different with the size of a firm. However, the best director on boards tends to be the CEO of bigger firm and the worst director is from the smaller firm. For instance, the highest-ability director on boards of Target is from the bigger firm than Target, but the lowest-ability one is the CEO of smaller company than Target.

Table 1-A Sample distribution of board composition I

Number of observations: 266 firms. All values are mean value. Total: total number of directors on boards, Out: total number of outside directors, and %: the proportion of outside directors on boards. Ongoing: the total number of other firm's ongoing CEO on boards. Retired: the total number of CEOs who retired from other firms on boards. SD is standard deviation.

	Total	Out	%	Ongoing	Retired
Mean	11.23	9.53	84.59	1.98	2.46
SD	2.14	2.24	9	1.52	1.48
Min	7	4	44.44	0	0
Max	20	17	100	10	8

Table 1-B Sample distribution of board composition II

Ongoing plus Retired: the total number of ongoing and retired CEOs on boards. %: the percentage of ongoing plus retired CEOs on boards. Ongoing plus Retired/Out (%): the proportion of ongoing plus retired CEOs in the total number of outside directors. SD is standard deviation.

	Ongoing plus Retired	%	Ongoing plus Retired/Out (%)
Mean	4.45	39.72	46.86
SD	2.08	17.42	20.2
Min	0	0	0
Max	11	88.89	100

Table 2-A Descriptive statistics for the average board quality

The average quality of boards of 266 firms among Fortune 500 U.S firms in 2005. All values are in $ thousand. Proxy for the average board quality: the compensation as CEOs, averaged over the retired CEOs plus ongoing CEOs on the same board. For the ongoing CEOs: the compensation as CEOs in 2004 year. For the retired CEOs: the converted level of compensations as CEOs one year before retirement. If A director retired in 2002, his quality is measured by (compensation as CEO in 2001)*(1+the median growth rate of CEO compensation between 2001 and 2004). The Wilcoxon rank-sum test shows that the mean of "The quality of boards: retired CEOs only" is significantly higher than the mean of "The quality of boards: ongoing CEOs only" at 1% level (***). The last column shows the dispersion of quality on the same boards which is measured by the ranking difference between the highest director and the lowest director on the same board. I rank 487 ongoing CEOs and former CEOs on boards of U.S. large firms in 2005 by the compensation levels as CEOs.

	The quality of boards	The quality of boards: ongoing CEOs only	The quality of boards: retired CEOs only	The dispersion of quality
Mean	11891.87	9849.937	14006.65***	226.88
SD	10274.73	7270.531	14087.53	114.06
Max	86896.84	39227.35	86896.84	463
Min	382.21	311	417	4
25%	6470.96	4743.48	6759.23	139
50%	9289.67	8420.43	10690.15	221.5
75%	14248.38	11590.36	15673.35	318

Table 2-B Descriptive statistics for quality of directors

	The ongoing CEOs on boards	The retired CEOs on boards
Number of observations	236	251
Mean	9683.26	13760.29***
Standard deviation	7689.18	16252.02

4.3 The average quality of boards: empirical result

Based on the equation (15),[27] the specification is given by

$$\overline{q_i}^* = G(S_i,\ u_i,\ \beta) + \epsilon$$

where $\overline{q_i}^*$ is the average board quality of firm i, S_i is the size of firm i and u_i is the prior expectation for quality of the incumbent CEO at the firm i (hereafter, the CEO quality). The proxies for the size of firm are (1) the market capitalization, (2) the amount of sales, and (3) the total assets. The proxy for the CEO quality is the CEO wage. *Table 3* shows the main outcome. The dependent variable in *Table 3* is the average quality of boards which is measured by the natural log value of the averaged compensations as CEO. The independent variables are the characteristics of firms at which ongoing CEOs and retired CEOs work as board members.[28] Overall, the talented candidates work (as outside directors) at firms with high sales.[29] In *Table 4*, I iterate the same regression with different proxy for the quality, market capitalization. The outcome is qualitatively similar as the outcome of *Table 3*. The talented people go to the firms with high market capitalization.

[27] We can not directly observe the cutoff quality level and minimum quality level. So, I use the quality of the highest one as a proxy for the cutoff quality level and of the lowest one as a proxy for the minimum quality level. Then, I regress the cutoff quality (the minimum quality) on the size of a firm and the prior expectation of incumbent CEO's talent. The size of a firm is positively correlated with the cutoff quality (minimum quality), which is predicted in this paper.

[28] I also run the regression including the industry dummy. The average board quality is lower at the bank industry.

[29] I also use the quantile regression to capture the different effects of the explanatory variables across the different quantile range of the board quality. The outcome is that: (1) sales are important, and (2) board compensation is positively significant only in the lower-middle quantile range. The qth quantile regression estimator $\hat{\beta}_q$ minimizes

$$\sum_{i:y_i \geq x_i'\beta}^{N} q \mid y_i - x_i'\beta \mid + \sum_{i:y_i \leq x_i'\beta}^{N} (1-q) \mid y_i - x_i'\beta \mid$$

See Cameron and Trivedi (2004) on page 87.

Table 3 How the average quality of boards depends on characteristics of firms

The dependent variable: the average quality of boards in 2005 measured by the natural log value of CEO compensations, averaged over ongoing CEOs plus retired CEOs on boards. All independent variables are the natural log value in 2004. The robust standard error is in parenthesis.

	OLS	OLS	OLS
CEO compensation		.10	.116*
		(.067)	(.066)
Market capitalization	-.03	-.051	.004
	(.106)	(.112)	(.131)
Total assets	.051	.043	.003
	(.099)	(.10)	(.121)
Sales	.258***	.253***	.225***
	(.082)	(.083)	(.084)
Annual director fee	.204*	.177	.17
	(.123)	(.124)	(.126)
Market to book ratio			.005
			(.009)
Long-term debt/total assets			.469
			(.431)
Constant	5.811***	5.242***	4.994***
	(.697)	(.832)	(.84)
R-squared	0.0893	0.1187	0.1197
N	184	183	179

Table 4 How the average quality of boards depends on characteristics of firms

The dependent variable: the quality of boards in 2005 measured by the natural log value of the market capitalization, averaged over ongoing CEOs plus retired CEOs on boards. All independent variables are the natural log value in 2004. The robust standard error is in parenthesis.

	OLS	OLS	OLS
CEO compensation		.143	.138
		(.136)	(.137)
Market capitalization	.635***	.585***	.66***
	(.244)	(.241)	(.243)
Total assets	-.255	-.243	-.308
	(.219)	(.222)	(.231)
Sales	.221	.221	.278
	(.169)	(.169)	(.17)
Annual director fee	.218	.188	.168
	(.182)	(.184)	(.185)
Market to book ratio			.013
			(.012)
Long-term debt/total assets			.703
			(.742)
Constant	3.24***	2.461	1.696
	(1.22)	(1.517)	(1.528)
R-squared	0.1668	0.1714	0.1931
N	199	198	194

4.4 Quality of a new board member: sample selection

The drawback of the previous approach is that we do not take into account the joining year of outside directors on boards, for instance, A director joins on boards of GE in 2005, B director in 2003, and C director in 2001. To mitigate this problem, I construct another data set to download the 2001-2005 board profiles of U.S firms which have total asset values greater than $million 1,000 in 2004 from The Investor Responsibility Research Center (IRRC) data base. Then, I select ongoing CEOs who newly join the boards as outside directors during 1995-2005.[30]

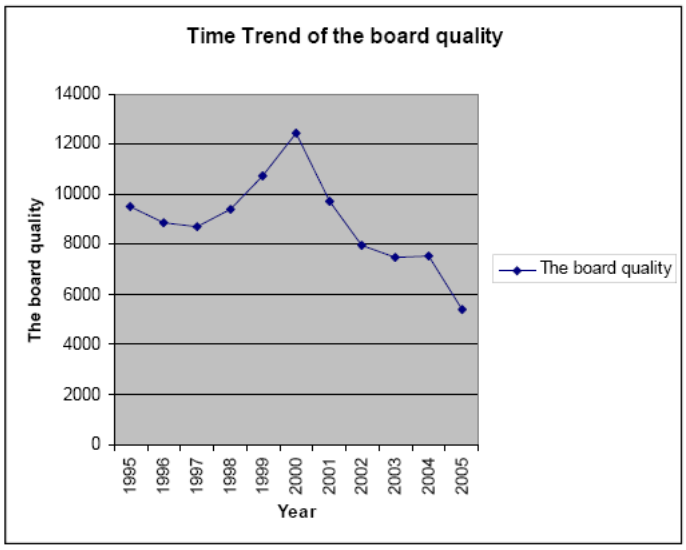

Figure 1: Time trend of the newly joined directors' quality

Both *Figure 1* and *Table 5* show the time trend of directors' quality during 1995-2005. These show that the quality of newly joined directors had risen during 1995-2000, but it shows the break in 2001. The quality had fallen during 2001-2005.[31]

4.5 Quality of a new board member: empirical result

As before, the specification is given by

$$q^*_{new,j} = G(S_i,\ u_i,\ \beta) + \epsilon$$

where $q^*_{new,j}$ is the quality of a newly joined director j on board i, S_i is the size of firm i and μ is the prior expectation for quality of the incumbent CEO at the firm i.

[30] In the regressions, I only use the data for the newly joined directors during 2000-2005 because of the data availability.

[31] My conjecture is that the talented potential candidates for outside directors might had escaped from the market for corporate directors due to Enron scandal and the increase in the liability of directors by SOX.

Table 5 Sample distribution of newly joined outside directors

I download 2001-2005 board profiles of U.S firms which have total asset values greater than $million 1,000 in 2004 from the IRRC data base. I select ongoing CEO's who newly join boards as outside directors during 1995-2005. The quality is measured by the CEO compensation one year before joining boards. The converted quality: I convert the quality measure into 2004 year value. For instance, if A ongoing CEO joins the boards in 2001, his quality is measured by (CEO compensation in 2000)*(1+the median growth rate of CEO compensation between 2000 and 2004) All values are mean values ($ thousand).

Joining year	N	The converted quality: the CEO compensation
1995	15	9499.27
1996	28	8872.37
1997	31	9017.83
1998	24	10761.86
1999	41	11024.35
2000	35	12455.85
2001	61	9716.44
2002	35	7487.426
2003	83	7951.995
2004	67	7507.653
2005	27	5377.355
Total	339	8636.49

Table 6 provides the outcome of the regression. The approach is the same as the previous regression in *Table 3*. In *Table 6-A and 6-B*, the dependent variable is the quality of the newly joined director j on board i measured by the CEO pay and the market capitalization of the CEO's original firm one year before joining the boards, respectively. Overall, the coefficients of the market capitalizations of firm i is positively significant. We can interpret that the talented ongoing CEOs go to firms with high market value to work as outside directors. Next, I test whether potential directors like to work at firms with talented CEO's or not. *Figure 2* shows the pairwise relationship between the board quality and the CEO quality. The quality of a newly joined director is measured by the CEO pay at his own firm and the quality of an incumbent CEO by the CEO pay at the firm i.

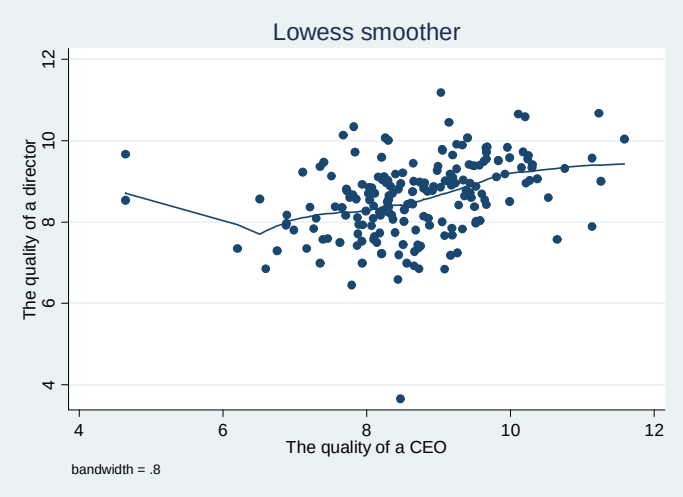

Figure 2: the relationship between the quality of a new director and the CEO quality

In the OLS estimations, the coefficient of the *Ln(CEO compensation)* is positively significant. The quality of a newly joined director is higher where the CEO pay is higher.[32] Finally, the point estimate indicates that board pay is not correlated with the quality of a newly joined director.

[32] However, there might be an endogeneity problem. There is academic discussion about the relationship between corporate governance (or board structure) and the CEO compensations. Core and Larcker (1999) suggest that the firms with weak board structures pay more to CEOs. Bebchuk and Fried (2003) argue that the current increase in CEO compensations can be explained by the increase in managerial entrenchment. Conversely, Hermalin (2005) suggests that the increase in CEO pay is due to tighter corporate governance. Gabaix and Landier (2008) provide evidence that the rise in CEO pay is partly due to the weak corporate governance, but the effect is relatively small. To test this, I use 2SLS. Based on Core and Larcker (1999), I use the dummy variable whether the CEO also takes the chairman position or not as the instrument for the CEO pay. I also use the tenure of CEO as the instrument. The market capitalization is still positively significant, but the effect of the incumbent CEO's talent is not significant.

Table 6-A How the quality of newly joined director i depends on characteristics of firm j: Ongoing CEO Directors only

I download 2001-2005 board profiles of U.S firms which have total asset values greater than $ million 1,000 in 2004 from the IRRC data base. I select ongoing CEOs who newly join boards as outside directors during 2000-2005. The dependent variable is the quality of a newly joined outside director measured by the natural log value of CEO pay at his original firm one year before joining boards. The standard error is in parenthesis.

	Pooled OLS	Pooled OLS	Pooled OLS
CEO compensation		.194**	.199***
		(.084)	(.087)
Market capitalization	.31***	.24*	.271
	(.129)	(.131)	(.17)
Total assets	-.131	-.099	-.117
	(.111)	(.111)	(.142)
Sales	.10	.069	.05
	(.097)	(.097)	(.104)
Annual director fee	.085	.033	.009
	(.164)	(.164)	(.173)
Market to book ratio			.0003
			(.03)
Long-term debt/total assets			-.09
			(.423)
Constant	5.589***	4.706***	4.809***
	(.551)	(.666)	(.713)
R-squared	0.2032	0.2248	0.2176
N	197	197	191

Table 6-B How the quality of newly joined director i depends on characteristics of firm j: Ongoing CEO Directors only

I download 2001-2005 board profiles of U.S firms which have total asset values greater than $ million 1,000 in 2004 from IRRC data base. I select ongoing CEOs who newly join boards as outside directors during 2000-2005. The dependent variable is the quality of a newly joined outside director measured by the natural log value for the market capitalization of his original firm. The standard error is in parenthesis.

	Pooled OLS	Pooled OLS	Pooled OLS
CEO compensation		.256**	.256**
		(.12)	(.122)
Market capitalization	.689***	.59***	.656***
	(.189)	(.193)	(.212)
Total assets	-.287*	-.246	-.255
	(.16)	(.16)	(.173)
Sales	.259*	.223	.169
	(.141)	(.141)	(.148)
Annual director fee	-.237	-.306	-.343
	(.242)	(.243)	(.251)
Market to book ratio			.018*
			(.009)
Long-term debt/total assets			.885
			(.666)
Constant	3.489***	2.386***	2.21**
	(.777)	(.928)	(.986)
R-squared	0.2840	0.2962	0.3006
N	270	270	262

4.6 Quality vs Experience

In this instance, an issue could be raised. We can have two possible theories to simply explain empirical evidence: (1) Ongoing and former CEO's of big companies make better directors for any company, and (2) Ongoing and former CEO's of big companies make good directors of big companies, but not small companies.[33] The argument of this paper is based on the former. In the second theory, the matching is consummated between the ongoing or former CEOs of big companies and the big companies because the experience of big companies' ongoing (former) CEOs is more valued only in the big companies' boards, which implies that the driving force behind the matching is not the quality, but the experience.[34] To test this, I adopt the following strategy: I calculate the predicted board quality based on the first OLS estimation in *Table 6-A*. Then, I measure the error term by

$$Error = y - y_{predicted}$$

The high error term indicates those firms which have ongoing CEOs of bigger companies as outside directors than expected. The Wilcoxon rank-sum test shows that the return on assets (ROA) of firms which are in the range of the upper 0%~25% error term is significantly higher than complements. Firms which perform well can get big firms' ongoing CEOs, which provides evidence that the driving force behind the match in the directorship market is not experience.

4.7 The director compensations

Based on the equation (16) and (17), I now regress the board compensations on the characteristics of firms. It follows that the basic empirical estimation is

$$\ln(W_{BOARD,i}) = \phi_1 \ln(S_i) + \phi_2 \ln(\mu_i) + \varepsilon$$

where $W_{BOARD,i}$ represents the board compensation of firm i, S_i is the size of firm i and μ_i is the prior expectation for the incumbent CEO at firm i. I download the board

[33] Eric Rasmusen enlightens me that we can have two different interpretations for the outcome of regressions.

[34] Konstantin Tyurin provides fruitful comments about this. He states that "to put it plainly, your theory makes, among other things, a testable prediction about the relationship between quality of the company board and the size of the company. So ultimately you are testing whether the size of the company where a given former (or ongoing) executive is a currently a board member is positively related to the average size of the companies where he served (is serving) as CEO in the past (now). This is exactly the matching story you're trying to explore in your theoretical part. However, the matching may have nothing (or little) to do with quality of CEOs, but rather be explained by the fact that experience accumulated as a CEO in a jumbo company would be more valuable if the same person serves on board of another jumbo company, and, conversely, experience accumulated as a CEO is a smaller-sized company would be more valuable if the same individual serves on board of another smaller-sized company upon retirement. Then you have changed the story that you're trying to test: it's nothing to do with quality but has a lot to do with finding a good match. To make an analogy with other markets (like the marriage market), it's not the quality that is driving the outcome but rather the driving force is a good match. In other words, smaller-size companies' CEOs and board members may not be inferior in quality, but simply the experience accumulated in such smaller companies may be different from the experience in super size companies. Then, it's not the quality that matters but the type of experience."

compensation data of U.S firms which have total asset values greater than $million 5, 000 in 2005. The board compensations are the annual director fees in 2005.[35] All characteristics of firms are 2004 year values. *Table 7* shows the descriptive statistics for the board compensation.[36]

Table 7 Descriptive statistics for the board compensations

I download the board compensations data of U.S firms which have total asset values greater than $ 5,000 million in 2005. The board compensations: annual director fees in 2005. All values are in $ thousand.

	The board compensation ($ thousand)
N	466
Mean	46.6
Standard deviation	20.64
Max	200
Min	6
25%	30
50%	45
75%	60

[35] The median annual director fee per board meeting is 7.28 in $ thousand. I calculate the annual director fee per board meeting by

$$\frac{\text{Median annual director fee}}{\text{Median number of board meeting}}.$$

The median number of board meetings is based on Linck, Netter and Yang (2006b). For a comparison, the median total cash compensation of CEO per day is 9.62 in $ thousand which is calculated by

$$\frac{\text{The median total cash compensation of CEO}}{\text{working day } (\approx 250)}$$

[36] You can find other statistics for the board compensation in Adams (2003) and Linck, Netter and Yang (2006b).

Table 8 The compensation for outside directors (Annual director fee): Cross-section data

I download financial information of U.S firms which have total asset values greater than $million 5,000 in 2005. The dependent variable is the annual directors' fees in 2005. The independent variables are the characteristics of firms. All values are 2004 year values.

	OLS	Quantile regression (.25)	Median quantile regression	Quantile regression (.75)
Asset	-.01	.007	-0.32	-.108*
	(.05)	(.104)	(.058)	(.06)
Sale	.66***	.926***	.704***	.399***
	(.14)	(.24)	(.132)	(.139)
Sale*Sale	-.03***	-.044***	-.029***	-.013*
	(.008)	(.013)	(.007)	(.008)
Market Capitalization	.001	.008	.026	.084
	(.05)	(.109)	(.060)	(.062)
CEO compensation	.13***	.129***	.099***	.12***
	(.03)	(.045)	(.025)	(.027)
ROA	-.0003	-.0002	.004	-.002
	(.004)	(.007)	(.004)	(.004)
Constant	-.88	-2.42**	-.911	.682
	(.67)	(1.122)	(.627)	(.659)
R-squared	0.3999			
Pseudo R-squared		0.2303	0.2305	0.2101
N	448	448	448	448

Table 8 provides OLS and quantile estimation results. Overall, the firms with high sales and with high-ability CEO pay more to board members. When the amount of sales increases by 1%, board compensations increase by 0.66%. However, the effect of sales is diminishing. Also, when the CEO compensation increases by 1%, the board compensation rises by 0.13%.

5 Conclusion

I construct a searching and matching model to explain the assignment pattern in the director market. I assume that (1) the managerial impact of CEO quality increases with the size of a firm under his control, (2) the CEO and the boards could be either complements or substitutes, in the production function of a firm, and (3) the directors enjoy reputation value on boards. This model suggests the matching pattern between the firms and potential directors under different circumstances. In general, the potential directors like to work as outside directors at firms with large size. The firms being big pay more and give more reputation value to directors. The matching pattern between CEOs and potential directors depends on (1) the interaction of both the agents, and (2) the definition of reputation value. When their reputation depends on the market value of a firm, they like to work at firms with talented CEOs whatever the CEO and boards are substitute or complement. However, when they consider "influence" or "importance" on boards as the reputation value, they would be at firms with low-ability CEOs if the CEO and the boards are substitutes. They feel more happy when they help lower-ability CEO's. Also, this model suggests the possibility that the quality of outside directors on the same boards could be very dispersed, which is consistent with the empirical findings. My estimates suggest that talented ongoing CEOs and retired CEOs go to those firms with high market capitalization to work as outside directors. It implies that market capitalization matter more as an indicator of the firm size than total assets and sales. The quality of boards is higher where the CEO pay is higher. I also find that firms with high sales pay more to outside directors and that the firms with highly paid CEOs pay more to outside directors, which implies that CEOs and boards are complements.

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6 Appendix : Proof

Proof. of proposition 1: For the simplicity, let $\sigma = 1$ in the equation (11). Taking a derivative of the boards pay, the equation (11), with respect to the size of a firm, we can get

$$\begin{aligned}\frac{\partial W_{BOARD}}{\partial S} &= \frac{\partial \left(2S\left[F(q^u)q^u - \mu\right]\overline{q_i} + S\mu\right)}{\partial S} \\ &= 2\left[F(q^u)q^u - \mu\right]\overline{q_i} + \mu\end{aligned}$$

Since we assume that $\overline{q_i} < \frac{-\mu}{2(F(q^u)q^u - \mu)}$, $\frac{\partial W_{BOARD}}{\partial S} > 0$. When we take a derivative with respect to the prior expectation of an incumbent CEO, it is given by

$$\frac{\partial W_{BOARD}}{\partial \mu} = \frac{\partial \left(2S\left[F(q^u)q^u - \mu\right]\overline{q_i} + S\mu\right)}{\partial \mu} = S(-2\overline{q_i} + 1)$$

Thus, if $\overline{q_i} < \frac{1}{2}$,then board compensation increases in the prior expectation of an incumbent CEO. ■

Proof. of Proposition 3: let me redefine the benefit, $B(q_{new}^{cut})$, and cost of an outside directorships, $C(q_{new}^{cut})$, by

$$B(q_{new}^{cut}) = E[\pi(q_c^k, q_{new}^{cut})] \ = y(q_{new}^{cut}) = C(q_{new}^{cut})$$

Since $\frac{\partial B(q_{new}^{cut})}{\partial q_{new}^{cut}} > 0$, $\frac{\partial^2 B(q_{new}^{cut})}{(\partial q_{new}^{cut})^2} < 0$, and $\frac{\partial C(q_{new}^{cut})}{\partial q_{new}^{cut}} > 0$, there exists a unique cutoff quality level which guarantees that potential candidates with $q_{new} \leq q_{new}^{cut}$ are likely to contact if $B(0) > C(0)$ and $B(1) < C(1)$. Taking a partial derivative of the benefit and cost of an outside directorships with respect to the size of a firm in "complement" case, we can get

$$\frac{\partial B(q_{new})}{\partial S} = \frac{\Omega\overline{q_i}^2 + S^\sigma \mu \overline{q_i}}{\partial S} \ and \ \frac{\partial C(q_{new})}{\partial S} = 0$$

It can be easily shown that $\frac{\partial B(q_{new})}{\partial S} > 0$, which implies that $\frac{\partial q_{new}^{cut}}{\partial S} > 0$. Similarly, we can find that $\frac{\partial q_{new}^{cut}}{\partial \mu} > 0$. By a similar method, it can easily be shown that $\frac{\partial q_{new}^{cut}}{\partial S} > 0$ and $\frac{\partial q_{new}^{cut}(\mu,S)}{\partial \mu} > 0$ in the "substitute" case ■

Proof. of Proposition 4: The left-hand side of the equation (18) is the cost of one more search denoted by $C(q_{new}^{\min})$. It can be easily shown that

$$\frac{\partial C(q_{new}^{\min})}{\partial q_{new}^{\min}} = \frac{\partial E[\pi(q_c^k, q_{new}^{\min})]}{\partial q_{new}^{\min}} > 0$$

The right-hand side represents the benefit of one more search denoted by $B(q_{new}^{\min})$. Taking

a derivative with respect to $q_{new}^{\min}$,

$$\begin{aligned}
&\frac{\partial B(q_{new}^{\min})}{\partial q_{new}^{\min}} \\
&= \gamma\left\{\frac{\frac{\partial\vartheta}{\partial q_{new}^{\min}}(1+\gamma)\left(E[\pi(q'_{new})-E[\pi(q_c^k,\overline{q_s})]\right)}{(\gamma+\vartheta)^2}\right\}+\gamma\left\{\frac{\vartheta\left(\frac{(1+\gamma)}{\gamma}\frac{\partial E[\pi(q'_{new})]}{\partial q_{new}^{\min}}\right)(\gamma+\vartheta)}{(\gamma+\vartheta)^2}\right\}<0
\end{aligned}$$

because $\frac{\partial\vartheta}{\partial q_{new}^{\min}}<0$ and $\frac{\partial E[\pi(q'_{new})]}{\partial q_{new}^{\min}}<0$. It is straightforward that $C(\overline{q_s})<B(\overline{q_s})$. So, if $C(q_{new}^{cut})\geq B(q_{new}^{cut})$, there exists a unique minimum quality level, $q_{new}^{\min}$, which guarantees that the firm would like to fill a vacancy with a potential candidate satisfying $q_{new}^{\min}\leq q_{new}\leq q_{new}^{cut}$. Next, we consider the effect of the firm size on the minimum quality level. Let me revisit the equation (18). Taking the total derivative of $C(q_{new}^{\min})$ and $B(q_{new}^{\min})$ with respect to the firm size, S,

$$\frac{dC(q_{new}^{\min})}{dS}=\frac{\partial E[\pi(q_c^k,q_{new}^{\min})]}{\partial q_{new}^{\min}}\left(\frac{dq_{new}^{\min}}{dS}\right)+\frac{\partial E[\pi(q_c^k,q_{new}^{\min})]}{\partial S}$$

and

$$\begin{aligned}
&\frac{dB(q_{new}^{\min})}{dS} \\
&= \gamma(1+\gamma)\left\{\frac{\frac{\partial E[\pi(q_c^k,\overline{q_s})]}{\partial S}(\gamma+\vartheta)}{(\gamma+\vartheta)^2}\right\}+\gamma(1+\gamma)\left\{\frac{\frac{\partial\vartheta}{\partial S}\left(E[\pi(q'_{new})]-E[\pi(q_c^k,\overline{q_s})]\right)}{(\gamma+\vartheta)^2}\right\} \\
&\quad -\gamma(1+\gamma)\left\{\frac{(\frac{1}{\gamma})\vartheta\left(\frac{\partial E[\pi(q'_{new})]}{\partial S}\right)(\gamma+\vartheta)}{(\gamma+\vartheta)^2}\right\}
\end{aligned}$$

At the minimum quality level, $B(q_{new}^{\min})\equiv C(q_{new}^{\min})$. Then,

$$\begin{aligned}
&\frac{dC(q_{new}^{\min})}{dS}-\frac{dB(q_{new}^{\min})}{dS} \\
&= \frac{\left\{[\gamma+(2+\gamma)\vartheta]\left[\left(\frac{\partial E[\pi(q'_{new})]}{\partial S}\right)\left(\frac{dq_{new}^{\min}}{dS}\right)+\frac{\partial E[\pi(q'_{new})]}{\partial S}\right]-\gamma(1+\gamma)\frac{\partial E[\pi(q_c^k,\overline{q_s})]}{\partial S}\right\}(\gamma+\vartheta)}{(\gamma+\vartheta_j)^2} \\
&\quad \frac{-\gamma(1+\gamma)\left[\left(\frac{dq_{new}^{cut}}{dS}\psi h(q'_{new})-\frac{dq_{new}^{\min}}{dS}\chi\psi h(q'_{new})\right)\left(E[\pi(q'_{new})]-E[\pi(q_c^k,\overline{q_s})]\right)\right]}{(\gamma+\vartheta_j)^2}=0
\end{aligned}$$

Finally, we can get

$$\frac{dq_{new}^{\min}}{dS}$$

$$= \frac{\gamma(1+\gamma)(\gamma+\vartheta)\frac{\partial E[\pi(q_c^k,\overline{q_s})]}{\partial S}+\gamma(1+\gamma)\left[\left(\frac{dq_{new}^{cut}}{dS}\psi h(q'_{new})\right)\left(E[\pi(q'_{new})]-E[\pi(q_c^k,\overline{q_s})]\right)\right]}{\left[(\gamma+(2+\gamma)\vartheta_j)\left(\frac{\partial E[\pi(q'_{new})]}{\partial q_{new}^{\min}}\right)(\gamma+\vartheta)+\gamma(1+\gamma)\psi h(q'_{new})\left(E[\pi(q'_{new})]-E[\pi(q_c^k,\overline{q_s})]\right)\right]}$$

$$-\frac{(\gamma+\vartheta)(\gamma+(2+\gamma)\vartheta)\frac{\partial E[\pi(q'_{new})]}{\partial S}}{\left[(\gamma+(2+\gamma)\vartheta)\left(\frac{\partial E[\pi(q'_{new})]}{\partial q_{new}^{\min}}\right)(\gamma+\vartheta)+\gamma(1+\gamma)\psi h(q'_{new})E[\pi(q'_{new})]-E[\pi(q_c^k,\overline{q_s})]\right]}$$

So, if

$$\frac{dq_{new}^{cut}}{dS} > \frac{(\gamma+\vartheta)(\gamma+(2+\gamma)\vartheta)\frac{\partial E[\pi(q'_{new})]}{\partial S}-\gamma(1+\gamma)(\gamma+\vartheta)\frac{\partial E[\pi(q_c^k,\overline{q_s})]}{\partial S}}{\gamma(1+\gamma)\left[\chi\psi h(q'_{new})\left(E[\pi(q'_{new})]-E[\pi(q_c^k,\overline{q_s})]\right)\right]},$$

then $\frac{dq_{new}^{\min}}{dS}>0$. Conclusively, the minimum quality level increases in the size of a firm if $\frac{dq_{new}^{cut}}{dS}$ is large enough. By a similar method, we can conclude that the minimum quality level increases in the CEO quality both in "complement" case and in "substitute" case $\frac{dq_{new}^{cut}}{d\mu}$ is large enough. ■